Mark Scheme Gravity Fields Past Paper Questions Jan 2002—Jan 2010 (old spec)

2

Q2 Jun 2003

(a) period = 24 hours or equals period of Earth's rotation ✓
remains in fixed position relative to surface of Earth ✓
equatorial orbit ✓
same angular speed as Earth or equatorial surface ✓ $_{\text{max}}(2)$

(b)(i) $\dfrac{GMm}{r^2} = m\omega^2 r$ ✓

$T = \dfrac{2\pi}{\omega}$ ✓

$r\left(= \dfrac{GMT^2}{4\pi^2}\right)^{1/3} = \left(\dfrac{6.67\times10^{-11}\times6\times10^{24}\times(24\times3600)^2}{4\pi^2}\right)^{1/3}$ ✓

(gives $r = 42.3 \times 10^3$ km)

(b)(ii) $\Delta V = GM\left(\dfrac{1}{R} - \dfrac{1}{r}\right)$ ✓

$= 6.67 \times 10^{-11} \times 6 \times 10^{24} \times \left(\dfrac{1}{6.4\times10^6} - \dfrac{1}{4.23\times10^7}\right) = 5.31 \times 10^7$ (J kg^{-1}) ✓

$\Delta E_p = m\Delta V$ $(= 750 \times 5.31 \times 10^7) = 3.98 \times 10^{10}$ J ✓
(allow C.E. for value of ΔV)

[alternatives:
calculation of $\dfrac{GM}{R}$ (6.25×10^7) or $\dfrac{GM}{r}$ (9.46×10^6) ✓
or calculation of $\dfrac{GMm}{R}$ (4.69×10^{10}) or $\dfrac{GMm}{r}$ (7.10×10^9)
calculation of both potential energy values ✓
subtraction of values or use of $m\Delta V$ with correct answer ✓] (6)

(8)

4

Q4 Jun 2004

(a) work = force × distance moved in direction of force ✓
(in circular motion) force is perpendicular to displacement ✓
no movement in direction of force ✓ (hence no work)
[or speed of body remains constant (although velocity changes) ✓
kinetic energy is constant ✓
potential energy is constant ✓]

[or gravitational force acts towards the Earth ✓
Moon remains at constant distance/radius from Earth ✓
since radius is unchanged, gravitational force does no work
or E_p of Moon is constant ✓] (*3*)

(b)(i) any suitable example of circular motion ✓

(ii) any SHM example at maximum displacement ✓
[or any other suitable example, e.g. car starts from rest] (*2*)
(*5*)

Quality of Written Communication: Q3(a) and Q4(a) ✓✓ (*2*)
(*2*)

Question 3

(a) work done/energy change (against the field) per unit mass ✓
when moved from infinity to the point ✓ (*2*)

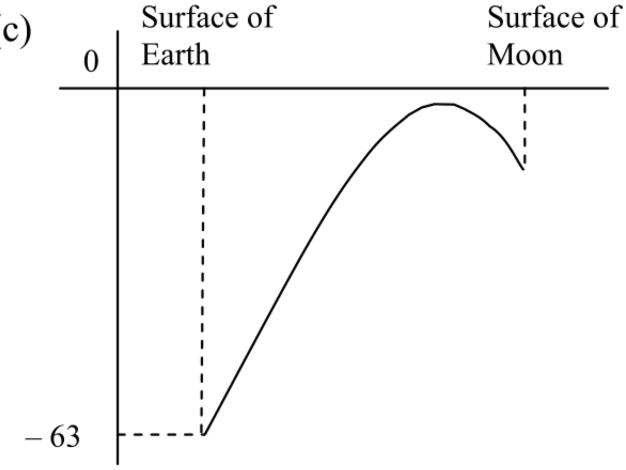
Q3 Jan 2005

(b) $V_E = -\dfrac{GM_E}{R_E}$ and $V_M = -\dfrac{GM_M}{R_M}$ ✓

$V_M = -\ G \times \dfrac{M_E}{81} \times \dfrac{3.7}{R_E} = \dfrac{3.7}{81} V_E$ ✓

$= 4.57 \times 10^{-2} \times (-63) = -2.9\ \text{MJ kg}^{-1}$ ✓ $(2.88\ \text{MJ kg}^{-1})$ (*3*)

(c)

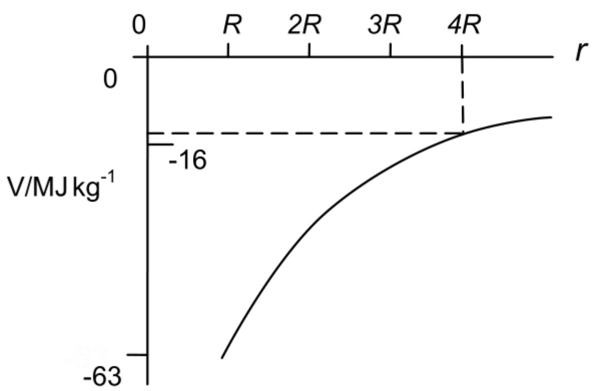


limiting values $(-63, -V_M)$ on correctly curving line ✓
rises to value close to but below zero ✓
falls to Moon ✓
from point much closer to M than E ✓

max(*3*)
(*8*)

Question 4	**Q4 Jun 2005**	
(a)	attractive **force** between point masses ✓ proportional to (product of) the masses ✓ inversely proportional to square of separation/distance apart ✓	**3**
(b)	$m\omega^2 R = (-)\dfrac{GMm}{R^2}\left(\text{or} = \dfrac{mv^2}{R}\right)$ ✓ (use of $T = \dfrac{2\pi}{\omega}$ gives) $\dfrac{4\pi^2}{T^2} = \dfrac{GM}{R^3}$ ✓ G and M are constants, hence $T^2 \propto R^3$ ✓	**3**
(c) (i)	(use of $T^2 \propto R^3$ gives) $\dfrac{365^2}{(1.50\times10^{11})^3} = \dfrac{T_m{}^2}{(5.79\times10^{10})^3}$ ✓ T_m = 87(.5) days ✓	**4**
(ii)	$\dfrac{1^2}{(1.50\times10^{11})^3} = \dfrac{165^2}{R_N{}^3}$ ✓ (gives $R_N = 4.52 \times 10^{12}$ m) ratio = $\dfrac{4.51\times10^{12}}{1.50\times10^{11}}$ = 30(.1) ✓	

Question 4	**Q4 Jan 2006**	
(a)	orbits (westwards) over Equator ✓ maintains a fixed position relative to surface of Earth ✓ period is 24 hrs (1 day) or same as for Earth's rotation ✓ offers uninterrupted communication between transmitter and receiver ✓ steerable dish not necessary ✓	**Max 3**
(b) (i)	$G\dfrac{Mm}{(R+h)^2} = m\omega^2(R+h)$ ✓	**6**
(ii)	use of $\omega = \dfrac{2\pi}{T}$ ✓ gives $\dfrac{GM}{(R+h)^3} = \dfrac{4\pi^2}{T^2}$, hence result ✓	
(iii)	limiting case is orbit at zero height i.e. $h = 0$ ✓ $T^2 = \left(\dfrac{4\pi^2 R^3}{GM}\right) = \dfrac{4\pi^2 \times (6.4\times10^6)^3}{6.67\times10^{-11}\times6.0\times10^{24}}$ ✓ T = 5090 s ✓ (= 85 min)	
(c)	speed increases ✓ loses potential energy but gains kinetic energy ✓ [or because $v^2 \propto \dfrac{1}{r}$ from $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$] [or because satellite must travel faster to stop it falling inwards when gravitational force increases]	**2**
	Total	**11**

Question 4		
(a)	force per unit mass ✓ [or force on a 1 kg mass or $g = F/m$ with terms explained] vector ✓	**2**
(b) (i)	$F\left(=\frac{GMm}{r^2}\right) = \frac{6.67\times10^{-11}\times6.00\times10^{24}\times2.5\times10^{3}}{\left(1.6\times10^{7}\right)^2}$ ✓ **Q4 Jan 2007** = 3900 N (3910) ✓	
(ii)	$V_{orbit}\left(=-\frac{GM}{r}\right) = -\frac{6.67\times10^{-11}\times6.00\times10^{24}}{1.6\times10^{7}}$ ✓ $= -25\ (\text{MJ kg}^{-1})\ (-25.0)$ ✓ [or $\frac{V_{orbit}}{V_{surface}} = \frac{r_{surface}}{r_{orbit}}$ ✓ gives $V_{orbit} = -\left(\frac{6.4\times10^{6}}{1.6\times10^{7}}\right)\times63 = -25\ (\text{MJ kg}^{-1})\ (-25.2)$ ✓] $\Delta V = (63 - 25)\times10^{6} = 38\times10^{6}\ (\text{J kg}^{-1})$ ✓ $\Delta E_p\ (= m\,\Delta V) = 2.5\times10^{3}\times38\times10^{6} = 9.5\times10^{10}$ J ✓	**max 5**
(c)	line starts at (R, –62.5) and ends at a finite value ✓ curve of decreasing positive gradient ✓ correct (1/r) relationship shown by axis values ✓ 	**3**
	Total	**10**

Question 5		
(a)	$\frac{GMm}{r^2} = m\omega^2 r$ (**or** = $\frac{mv^2}{r}$) ✓ **Q5 Jan 2008** correct application of $T = \frac{2\pi}{\omega}$ (**or** $v = \frac{2\pi r}{T}$) ✓ (gives $\frac{GMm}{r^2} = \frac{4\pi^2 mr}{T^2}$ and $T^2 = \frac{4\pi^2 r^3}{GM}$) in which m has cancelled or does not appear in expression ✓	**3**
(b) (i) (ii) (iii)	$\omega \left(= \frac{2\pi}{T}\right) = \frac{2\pi}{7.15 \times 24 \times 3600}$ = 1.0(2) × 10^{-5} (rad) s^{-1} ✓ $\omega^2 r$ $(1.02 \times 10^{-5})^2 \times 1.07 \times 10^9$ = 0.11(1) m s^{-2} ✓ centripetal acceleration = g (**or** $\alpha = \frac{GM}{r^2}$) ✓ $M \left(= \frac{gr^2}{G}\right) = \frac{0.111 \times (1.07 \times 10^9)^2}{6.67 \times 10^{-11}}$ ✓ = 1.9(1) × 10^{27} kg ✓ [**or** use of $T^2 = \frac{4\pi^2 r^3}{GM}$ with T = 6.18 × 10^5 s ✓ gives $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 \times (1.07 \times 10^9)^3}{6.67 \times 10^{-11} \times (6.18 \times 10^5)^2}$ ✓ = 1.9(0) × 10^{27} kg ✓]	**max 5**
	Total	**8**

Question 3		
(a) (i)	gravitational force (or field) decreases as r increases ✓	
	gravitational force (or field strength) is proportional to $(1/r^2)$ ✓	
	[award both marks for second statement alone] **Q3 Jun 2008**	
(ii)	mass of Moon M $\left(= \frac{Fr^2}{Gm}\right) = \frac{1600 \times (1.75 \times 10^6)^2}{6.67 \times 10^{-11} \times 1000}$ ✓	4
	$= 7.3(5) \times 10^{22}$ kg ✓	
	[**or** by use of any other consistent values of F and r]	
(b) (i)	E_P lost = area under graph ✓	
	acceptable method for finding area and values ✓	
	acceptable value for E_P lost ✓ [allow $(2.8 \pm 1.0) \times 10^9$ J]	
	[**alternative mark scheme**, for candidates who use values from the graph:	
	potential of Moon's surface	
	$= \left(-\frac{GM}{r}\right) = -\frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{1.75 \times 10^6} = -2.80 \times 10^6$ (J kg^{-1}) ✓	
	change in potential $\Delta V = (-2.80 \times 10^6) - 0$	
	$= (-)2.80 \times 10^6$ (J kg^{-1}) ✓	
	potential energy lost ($= m\,\Delta V$) $= 1000 \times 2.80 \times 10^6$	5
	$= 2.80 \times 10^9$ J ✓]	
(ii)	$½mv^2 = 2.8 \times 10^9$ (or the E_P value from (b) (i)) ✓	
	gives escape speed $v = 2370$ m s^{-1} (or a consistent value) ✓	
	[**alternative mark scheme**, for candidates who use gravitational potential equation:	
	$½mv^2 = \frac{GMm}{r}$ gives $v = \sqrt{\frac{2GM}{r}}$	
	$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{1.75 \times 10^6}}$ ✓	
	$= 2370$ m s^{-1} ✓]	
	Total	**9**

Question 3		
(a) (i)	**Q3 Jun 2009** gradient $\left(= \frac{5.9\times10^8}{6000}\right)$ = 9.83×10^4 ($m\,s^{-2/3}$) ✓ (for 9.83 allow 9.7 to 10.0)	
(ii)	cube root of equation is $R = \left(\frac{GM}{4\pi^2}\right)^{1/3} T^{2/3}$ (or equation predicts $R \propto T^{2/3}$) ✓ $R \propto T^{2/3}$ confirmed by graph as a straight line through (0, 0) (or a line of constant gradient through (0, 0)) ✓	6
(iii)	use of gradient of graph as $\left(\frac{GM}{4\pi^2}\right)^{1/3}$ or $\left(\frac{R}{T^{2/3}}\right)$ ✓ $\left(\frac{GM}{4\pi^2}\right)^{1/3}$ = 9.83×10^4 gives $\left(\frac{GM}{4\pi^2}\right)$ = 9.50×10^{14} ($m^3\,s^{-2}$) ✓ mass of Saturn $M = \frac{9.50\times10^{14}\times4\pi^2}{6.67\times10^{-11}}$ = 5.62×10^{26} kg ✓	
(b)	similarity: graph would also be a straight line (through (0, 0) because $R \propto T^{2/3}$ (or $R^3 \propto T^2$) always applies to any satellite ✓ difference: gradient would be *larger* because mass of Sun > mass of Saturn ✓	2
	Total	**8**